Hierarchical Multimodel Saltwater Intrusion Remediation and Sampling Designs: A BMA Tree Approach

Basic Information

<table>
<thead>
<tr>
<th>Title:</th>
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Publications


12. • Frank Tsai, 2012, Feasibility Study of Scavenging Approach to Stop Saltwater Toward Water Wells, Louisiana State University, Baton Rouge, Louisiana, 10 pages. (USGS 104B)


16. • Frank Tsai, 2012, Feasibility Study of Scavenging Approach to Stop Saltwater Toward Water Wells, Louisiana State University, Baton Rouge, Louisiana, 10 pages. (USGS 104B)


25. Frank Tsai, 2012, Feasibility Study of Scavenging Approach to Stop Saltwater Toward Water Wells, Louisiana State University, Baton Rouge, Louisiana, 10 pages. (USGS 104B)

26. Frank Tsai, 2013, Hydrostratigraphy Modeling of the Southern Hills Aquifer System and Faults, Louisiana State University, Baton Rouge, Louisiana, 10 pages. (USGS 104B)


42. • Frank Tsai, 2012. Feasibility Study of Scavenging Approach to Stop Saltwater Toward Water Wells, Louisiana State University, Baton Rouge, Louisiana, 10 pages. (USGS 104B)


51. Frank Tsai, 2012, Feasibility Study of Scavenging Approach to Stop Saltwater Toward Water Wells, Louisiana State University, Baton Rouge, Louisiana, 10 pages. (USGS 104B)
52. Frank Tsai, 2013, Hydrostratigraphy Modeling of the Southern Hills Aquifer System and Faults, Louisiana State University, Baton Rouge, Louisiana, 10 pages. (USGS 104B)
58. Pham, H. V. and F. T.-C. Tsai, Development of groundwater model for the “1,200-foot”, “1,500-foot” and “1,700-foot” sands of the Baton Rouge area, Southeastern Louisiana, 7th Annual Groundwater and Water Resources Symposia, Baton Rouge, LA, 8 May 2013


78. Frank Tsai. 2013, Hydrostratigraphy Modeling of the Southern Hills Aquifer System and Faults, Louisiana State University, Baton Rouge, Louisiana, 10 pages. (USGS 104B)
Problem and Research Objectives
Water use in Baton Rouge, Louisiana is approximately 171.41 million gallons per day out of which 87.4% is ground water and the rest is surface water (Sargent, 2012). Population served by public supply is 436,650. Due to excessive ground water pumping, saltwater is intruding from the saline aquifers in the south part of the Baton Rouge Fault. Thus, in the absence of any remediation measure, some of public supply water wells in East Baton Rouge Parish are under the threat of being abandoned in the near future. The project objective is to develop saltwater intrusion models to be employed for the management and remediation of the ground water resources for the study area shown in Figure 1. The study area is approximately 500 km².

Figure 1: The map of the study area. Circles are pumping wells. White triangles are USGS water wells. All wells in the map were screened at the “1,500-foot” sand.

Due to limited amount of data and since model uncertainty always exists, multiple models are usually developed. Model selection, model elimination, model reduction, and model discrimination are commonly used to select the best model. It is clear that modeling uncertainty is always underestimated if only the best model is used. One would ask why only the best model is used afterwards when so many efforts have been devoted to calibrating many models. This certainly wastes valuable resources and important information from other good models. Hierarchical Bayesian model averaging (HBMA) (Chitsazan and Tsai, 2012; Tsai and Elshall, 2013) best utilize all possible models for model prediction and application under Bayesian statistical framework. HBMA presents several advantages over model selection: (1) Information from all possible models is used based on their model importance (model weights). Calibration efforts are not wasted. (2) The model importance is based on the evidence of data, which avoids over-confidence in the best model that does not have a dominant model weight. And (3) model structure uncertainty is increased and is better presented than that by using a single model. Moreover, HBMA is able to distinguish model uncertainty arising from individual models and
between models. HBMA is able to identify unfavorable models even though they may present small prediction uncertainty.

In this study, the HBMA is used to predict chloride concentration and estimate prediction uncertainty for the “1,500-foot” sand in the Baton Rouge aquifer system. The HBMA is applied to analyze the conceptual model structure uncertainty arising from the different competing model propositions for chloride concentration predictions at the USGS water quality wells.

**Methodology**

**Hierarchical Bayesian Model Averaging (HBMA)**

To cope with different sources of uncertainty in groundwater flow and mass transport models, a hierarchical Bayesian model averaging is developed (Tsai and Elshall, 2013). Consider \( M_{(ij...lm)}^p \) a model at level \( p \). The subscript \( (ij...lm) \) locates the model hierarchically top down from the first level, to the second level and so forth to reach to level \( p \). For example, \( M_{(i)} \) is model \( i \) at level 1, \( M_{(ij)} \) is model \( j \) at level 2, which is a child model to parent model \( i \) at level 1. \( M_{(ijk)} \) is model \( k \) at level 3, which is a child model to the parent model \( j \) at level 2 and the grandparent model of model \( i \) at level 1. From bottom up, parent models \( M_{p-1} \) at level \( p-1 \) is composed of the child models \( M_p \) at level \( p \). Models \( M_{p-2} \) at level \( p-2 \) are composed of models \( M_{p-1} \) at level \( p-1 \) and so forth until the Hierarch BMA model \( M_0 \) is reached.

Consider base models at level \( p \). According to the law of total probability, the posterior probability for predicted quantity \( \Delta \) given data \( D \) is

\[
Pr(\Delta | D) = E_{M_1} E_{M_2} ... E_{M_p} \left[ Pr(\Delta | D, M_p) \right],
\]

(1)

where \( E_{M_p} \) is the expectation operator with respect to models \( M_p \) at level \( p \). \( Pr(\Delta | D, M_p) \) is the posterior probability of predicted quantity \( \Delta \) given data \( D \) and models \( M_p \) at level \( p \). The expectation \( E_{M_p} \left[ Pr(\Delta | D, M_p) \right] \) is posterior probability averaging at level \( p \). That is

\[
E_{M_p} \left[ Pr(\Delta | D, M_p) \right] = \sum_{M_p} \Pr \left( \Delta | D, M_{(i|j|k)...}^p \right) \Pr \left( M_{(i|j|k)...}^p | D, M_{(i|j|k)...}^{p-1} \right).
\]

(2)

where \( \Pr \left( \Delta | D, M_{(i|j|k)...}^p \right) = \Pr(\Delta | D, M_p) \).

\( \Pr \left( M_{(i|j|k)...}^p | D, M_{(i|j|k)...}^{p-1} \right) = \Pr(M_p | D, M_{p-1}) \) is the conditional posterior model probability of model \( M_{(i|j|k)...}^p \) at level \( p \) under model \( M_{(i|j|k)...}^{p-1} \) at level \( p-1 \). \( \Pr(M_p | D, M_{p-1}) \) also represents the conditional model weights and will be used to develop a BMA tree of model weights. Note that model \( M_{(i|j|k)...}^p \) is a child model under the parent model \( M_{(i|j|k)...}^{p-1} \) because both have the same subscript for the first \( p-1 \) levels. Equation (2) is the Bayesian model averaging (BMA) at level \( p \), which can be written as
\[
\Pr(\Delta | D, M_{p-1}) = E_{M_p}[\Pr(\Delta | D, M_p)].
\]

(3)

According to equations (1) and (3), one can derive the posterior probability of prediction using BMA over models at any level, say level n:

\[
\Pr(\Delta | D, M_n) = E_{M_{n-1}} E_{M_{n+1}} \cdots E_{M_p} \Pr(\Delta | D, M_p).
\]

(4)

Based on equation (4), the law of total expectation and the law of total variance, the prediction mean, within-model variance, between model variance and total variance can be derived at level n.

The hierarchy BMA model is the usual BMA model (Hoeting et al., 1999), which is based on equation (1). The hierarchy model obtains model averaging results and prediction variances using all base models.

In this study, \( \Delta \) is the concentration and \( D \) is groundwater head and concentration data used to calibrate groundwater flow and transport models.

**Principal Findings and Significance**

(1) **Saltwater intrusion modeling in the “1,500-foot” sand of the Baton Rouge area**

We develop a two-dimensional groundwater flow and mass transport model to predict the saltwater intrusion in the “1,500-foot” sand of the Baton Rouge area. The study area, shown in Figure 1, includes the east-west trending Baton Rouge fault (see Figure 1). The saltwater intrusion model in this report is based on Tsai (2010, 2011). The simulation period is from 1/1/1990 to 12/31/2029 which is divided in calibration part from 1/1/1990 to 1/1/2005 and prediction part from 1/1/2005 to 12/31/2029. The initial groundwater head and the initial chloride concentration are obtained from Tsai (2011). The groundwater model uses the time-varied constant boundary condition for all the boundaries. The mass transport model uses constant concentration in the south boundary. The concentrations in the other boundaries are calculated by the transport simulation model in each time step. The major production wells are Lula pump station and Government Street pump station, which are located north of the Baton Rouge fault. The average pumping rate from Lula pump station is 7.03 million gallons per day and at Government Street pump station is 1.59 million gallons per day. We use MODFLOW (Harbaugh, 2005) and MT3DMS (Zheng and Wang, 1999) to simulate the groundwater flow and mass transport from 1/1/1990 to 12/31/2029. We use 706 head observations from 1/1/1990 to 1/1/2005 at the USGS observation wells shown in Figure 1 to calibrate the model. Then, we develop the prediction models to predict salt water intrusion from 1/1/2005 to 12/31/2029.

(2) **Sources of uncertainty and multiple models**

We analyze four sources of uncertainty in a hierarchical order in the flow and transport models. They are (1) boundary condition uncertainty, (2) grain-size method uncertainty in determining point-wise hydraulic conductivity, (3) variogram model uncertainty in kriging hydraulic conductivity distribution, and (4) fault permeability architecture uncertainty. To address these sources of uncertainty, 5 boundary condition propositions, three grain-size methods (Kozeny-Carman, Slitzer, and Terzaghi methods), three variogram models (exponential, Gaussian, and spherical), and 3 fault permeability architectures are proposed. This results in \( 5 \times 3 \times 3 \times 4 = 180 \) saltwater intrusion simulation models at the base level of the BMA tree.
In order to track a model in the BMA tree, we use the letter “B” subscribed with percentage of change of boundary head values in the determined boundary condition, the first letter of the grain-size methods, the first letter of the variogram model and the number of fault permeability segments in a hierarchical way to denote a model. For example “B_3KG3” denotes a base model in level 4 that consider no change in the determined boundary condition, Kozeny-Carman method, the Gaussian variogram and three-segment fault permeability architecture. “B_3KG” is a BMA model at level 3 that averages base models with different fault permeability architectures given “B_3” boundary condition, “K” grain-sized method and “G” variogram model propositions. “B_0K” is a BMA model at level 2 that averages level-3 BMA models with different grain-size method propositions given “B_0” boundary condition proposition. “B_0” is a BMA model at level 1 that averages level-2 BMA models with different variogram model propositions given “B_0” boundary condition and “K” grain-sized method propositions.

(3) BMA Tree of model weights

Figure 2 shows the BMA tree of model weights in parentheses and conditional model weights. The model weights reflect the comparative importance of all the competitive modeling propositions in one level. The conditional model weights represent the relative importance of the different propositions under the same parent models. The base level of the BMA tree corresponds to different fault permeability segments. The simulation models using homogeneous fault permeability can be discarded because they provide very poor fitting to the observation data and are not shown in the BMA tree.

At the base level, the best base model is B_0KG3 with the model weight 20.41%. At the third level, the BMA models are developed by averaging concentration predictions from their child base models that use different fault permeability architectures. The “B_0KG” is the best model with model weight 38.93% and the second best model is the “B_0KS” with model weight 28.75%. The relative model weights show that the ranking of the variogram models is the same under both “B_0K” and “B_0+10K” models. The Gaussian model is a better proposition than the spherical and exponential models to determine the hydraulic conductivity distribution.

At the second level, BMA models are developed by averaging concentration predictions from their child BMA models that use different variogram models for hydraulic conductivity estimation. As shown in Figure 2, since Terzaghi and Slichter methods have significantly worse fit to the observation data, only Kozeny-Carman method are remained at the second level. The “B_0K” model weight is 78.64% and the “B_0+10K” model weight is 21.36%. Their conditional model weights are 100% under their parent model.

At the first level, BMA models are developed by averaging concentration predictions by their child BMA models that use different grain-size methods. However, from previous analysis, we found that only Kozeny-Carman method was left to be used. At this level the determined boundary condition (B_0) is dominantly the best model with the model weight 78.64% and B_0+10 is the second best model with model weight 21.36%. Other boundary condition propositions are discarded because of poor fitting to the observation data.
(4) BMA tree of mean concentration predictions

The BMA tree of mean concentration predictions for the EB-658 and the EB-917 on 12/31/2029 is shown in Figure 3. The predicted mean concentration at EB-658 at the base level is between 481.76 mg/L and 1110.91 mg/L and. The predicted mean concentration at EB-917 at the base level is between 3816.20 mg/L and 4747.73 mg/L. The mean concentration prediction range becomes narrower while going up to upper layers because of the nature of averaging and the reduction in the number of models. At the level 3 the mean concentration range at EB-658 is between 527.29 mg/L and 926.38 mg/L and the mean concentration range at EB-917 is between 3995.03 mg/L and 4495.88 mg/L. At the level 2 and level 3, the concentration prediction range for EB-658 is between 594.90 mg/L and 835.68 mg/L and for EB-917 is between 4181 mg/L and 4436.23 mg/L. The hierarchy BMA model predicts mean concentration at EB-658 and EB-917 is 646.33 mg/L and 4236.04 mg/L. It was found that all of the models with 3-segment fault proposition predict higher mean concentration at EB-917 than the models with 2-segment or 4-segment fault proposition.

The BMA tree of mean predictions shown in Figure 3 provides an understanding of mean prediction variability over the accumulation of sources of uncertainty, which is not possible to know via the traditional BMA method.
Figure 3: BMA tree of mean concentration predictions (mg/L) at the EB-658 and EB-917 at the 12/31/2029.

(5) Temporal predictions and variances
Figure 4 shows the EB-917 concentration predictions and the one standard deviation bound using models at the different levels for the prediction period. All of the models predict the increasing concentration at EB-917. As shown in Figure 4, none of the USGS chloride data is inside the one standard deviation bound of the B0KG3 base model and B0KG model. Two chloride data are in one standard deviation bound of the B0K and B0 models and all chloride data are in the one standard deviation bound of the hierarch model.

According to Figure 4, it is clear to see that prediction variance caused by uncertain model parameters is much smaller than that caused by different model propositions. Moreover, the prediction variances at all levels start to increase at the beginning of time and then decrease. This behavior is reasonable because at early time all models predict similar low concentration at EB-917. Therefore the prediction variances are small. High prediction variances occur due to predicting concentration quite differently by different models. At later time all models start to predict similar high concentration at EB-917. Therefore, prediction variance decreases. The hierarch model has much higher prediction variance comparing to the B0 model is because it includes high prediction variance from the B10 model.
Figure 4: One standard deviation error bound of predicted concentration at EB-917 from 1/1/2005 to 12/31/2029. Crosses are USGS chloride data.

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Tsai, F.T.-C. 2011. Scavenger well operation model to assist BRWC to identify cost-effective approaches to stop saltwater intrusion toward the BRWC water wells in the “1,500-foot” sand of the Baton Rouge area. Technical report to Baton Rouge Water Company and Capital Area Groundwater Conservation Commission, Louisiana.

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