# Multimodel uncertainty analysis for chance-constrained saltwater intrusion management

### **Basic Information**

| Title:                         | Multimodel uncertainty analysis for chance-constrained saltwater intrusion management |
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| Project Number:                | 2010LA71B   |
| <b>Start Date:</b>             | 3/1/2010  |
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| Principal<br>Investigators:    | Frank Tsai  |

# **Publications**

- 1. Tubbs, K. R., and F. T.-C. Tsai. (2010). GPU Accelerated Lattice Boltzmann Model for Shallow Water Flow and Mass Transport, International Journal for Numerical Methods in Engineering, DOI: 10.1002/nme.3066.
- 2. Tubbs, K. R., and F. T.-C. Tsai. (2010). GPU Accelerated Lattice Boltzmann Model for Shallow Water Flow and Mass Transport, International Journal for Numerical Methods in Engineering, DOI: 10.1002/nme.3066.
- 3. Servan-Camas, B., and F. T.-C. Tsai. (2010). Two-Relaxation-Time Lattice Boltzmann Method for Anisotropic Dispersive Henry Problem. Water Resources Research. doi:10.1029/2009WR007837.
- 4. Tubbs, K. R., and F. T.-C. Tsai (2009). Multilayer Shallow Water Flow using Lattice Boltzmann Model with High Performance Computing. Advances in Water Resources, 32(11), 1767-1776.
- 5. Li, X., and F. T.-C. Tsai.(2009). Bayesian Model Averaging for Groundwater Head Prediction and Uncertainty Analysis Using Multimodel and Multimethod. Water Resources Research, 45, W09403. doi:10.1029/2008WR007488.
- 6. Tsai, F. T.-C., Bayesian Model Averaging Assessment on Groundwater Management under Model Structure Uncertainty, Stochastic Environmental Research and Risk Assessment, 24(6), 845-861, 2010.
- 7. Tsai, F. T.-C. and W. W-G. Yeh. (2010). Chapter 7: Model Calibration and Parameter Structure Identification in Characterization of Ground Water Systems, in Ground Water Management Manual (M. Aral and S. Taylor ed.) American Society of Civil Engineers. Accepted.
- 8. Kevin R. Tubbs, 2010, Ph.D. Dissertation "Lattice Boltzmann Modeling for Shallow Water Equations Using High Performance Computing, Engineering Science Program, College of Engineering, Louisiana State University, Baton Rouge, Louisiana, 128 pages.
- 9. Tsai, F. T.-C. (2010). A Co-Generalized Parameterization Method for Hydraulic Conductivity Estimation, World Water & Environmental Resources Congress, Providence, Rhode Island, May 16-20, 2010.
- Tsai, F. T.-C. (2010). Multimodel Approach for Groundwater Model Calibration, Prediction, and Application, World Water & Environmental Resources Congress, Providence, Rhode Island, May 16-20, 2010. (invited)

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- 11. Tsai, F. T.-C. (2010). Data Fusion using Co-Generalized Parameterization: Hydraulic Conductivity Estimation, 2010 Western Pacific Geophysics Meeting, 22–25 June 2010, Taipei, Taiwan
- 12. Tsai, F. T.-C. (2010). Hierarchical Bayesian Model Averaging for Groundwater Multimodel Prediction and Management under Uncertainty, 2010 Western Pacific Geophysics Meeting, 22–25 June 2010, Taipei, Taiwan
- 13. Tsai, F. T.-C. (2010). A Co-Generalized Parameterization Method for Hydraulic Conductivity Estimation, World Water & Environmental Resources Congress, Providence, Rhode Island, May 16-20, 2010.
- 14. Tsai, F. T.-C. (2010). Multimodel Approach for Groundwater Model Calibration, Prediction, and Application, World Water & Environmental Resources Congress, Providence, Rhode Island, May 16-20, 2010. (invited)

Publications 2

#### **SYNOPSIS**

Title: Multimodel Uncertainty Analysis for Chance-Constrained Saltwater Intrusion

Management Project Number: Start Date: 3/1/2010 End Date: 2/28/2011 Funding Source: 104B

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**Research Category:** Ground-water Flow and Transport

Focus Category: Ground Water, Solute Transport, Management & Planning

Descriptors: Uncertainty, Chance Constraint, Model Averaging, Prediction, Saltwater Intrusion,

Aquifer, Subsurface, Modeling, Optimization, Management

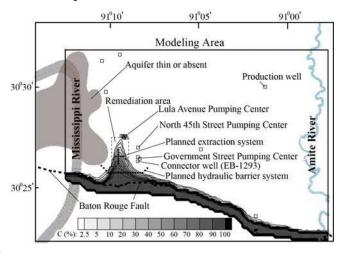
Primary PI: Frank T.-C. Tsai

#### **Problem and Research Objectives**

The Baton Rouge aquifer system located at south central Louisiana is a major source of drinking and industrial water. The aquifer has a fault running east-west located at the southern part near the coastline of the region. The fault cuts the aquifer system into two parts: the upthrown north side and the down-thrown south side. The fault was considered to act as an impermeable barrier to groundwater movement across it. A recent study suggests the Baton Rouge Fault as a conduit-barrier fault (Bense and Person 2006). Predominantly, the region south of the aquifer contains saltwater and north of the aquifer contains fresh water. However, by 1990, the water quality data at the existing wells to the north of the fault indicated that increasing water withdrawn in the region was resulting in saltwater intrusion to the north and a decrease in water quality within the aquifer system (Tomaszewski 1996). The sources of the saltwater are nearby the St. Gabriel salt dome and Darrow salt dome (Bray and Hanor 1990). The project focuses on mitigating saltwater intrusion in the "1,500-foot" sand aquifer.

**Figure 1:** The study area of the "1,500-foot" sand aquifer in the Baton Rouge area, Louisiana. The contour lines represent the saltwater concentration (%) distribution at the beginning of 2005.

The study area shown in Figure 1 is the "1,500-foot" sand aquifer, where the extent of the saltwater intrusion was predicted at the beginning of year 2005. There are three major groundwater production centers in this area, which have developed a large depression cone and caused saltwater intrusion from south of the Baton Rouge fault. Recent study of



groundwater modeling in this area indicated that the groundwater heads are continuously decreasing (Tsai and Li 2008a, Li and Tsai 2009, Tsai 2010), which could result in undesired chloride concentration at production wells in the future.

The objective of the study is to develop a management model using an injection-extraction approach to protect the production wells from saltwater intrusion. The idea has been actually implemented for hydraulic control to the West Coast Basin of coastal Los Angeles, California (Reichard and Johnson 2005) and was considered in Spain (Abarca et al. 2006). This study considers the joint operations of the hydraulic barrier system and the extraction system shown in Figure 1 to (i) intercept the incoming saltwater plume toward the production wells and (ii) reduce brackish water north of the fault. The injection wells align to form a hydraulic barrier to reduce saltwater movement towards the production wells. The pumping wells are placed at the pathway of the brackish water in order to remove the brackish water from the aquifer and prevent northward movement of the brackish water pushed by the hydraulic barrier system. The locations of these well pumps are fixed in this study.

#### Methodology

#### (1) Genetic Algorithm for Injection-Extraction Management Model

The overriding objective of the management model is to minimize the total amount of injected and extracted water as follows

$$\min_{\substack{z_{i,n}^R \in \{0,1\}, q^R \\ z_{j,n}^P \in \{0,1\}, q^P}} \sum_{i} \sum_{n} z_{i,n}^R q^R \Delta t_n + \sum_{j} \sum_{n} z_{j,n}^P q^P \Delta t_n ,$$
(1)

The range of injection and extraction rates is constrained by

$$0 \le q^R \le q_{\text{max}}^R,$$

$$0 \le q^P \le q_{\text{max}}^P,$$

$$(2)$$

where  $q^R$  and  $q^P$  are the injection rate and the extraction rate, respectively.  $q_{\max}^R$  and  $q_{\max}^P$  are the maximum injection rate and extraction rate, respectively.  $z_{i,n}^R$  and  $z_{j,n}^P$  are the scheduling binary variables for spatial and temporal allocation of the pump rates at injection site i, pumping site j, at time period n.  $\Delta t_n$  is the time interval for the period n. To reduce operation complexity, this study searches for optimal constant injection rate and constant extraction rate and optimal operation schedule to determine well pump activities.

The concentration at the Lula Avenue pumping center (see Lula wells in Figure 1) is constrained by the maximum permissible level (MPL):

$$C\left(\mathbf{x} = \mathbf{x}_{tuta}, t \in [t_0, t_T]; z_{i,n}^R, q^R, z_{i,n}^P, q^P\right) \le C_{MPL},\tag{3}$$

where C is the predicted concentration by the simulation models,  $C^{tar}$  is the maximum permissible level (MPL) of concentration,  $\mathbf{x}_{Lula}$  is the location of Lula wells,  $t_0$  is the starting time of remediation horizon, and  $t_T$  is the ending time of the remediation horizon. The concentration in the remediation area is also constrained by the MPL:

$$C\left(\mathbf{x} \in \Omega_{R}, t = t_{T}; z_{i,n}^{R}, q^{R}, z_{j,n}^{P}, q^{P}\right) \le C_{MPL}, \tag{4}$$

where  $\Omega_R$  is the domain of remediation area (see Figure 1). The joint operations of hydraulic barrier and extraction systems present a mixed integer nonlinear programming (MINLP) problem, which involves the ground water model and transport model. This study employs a genetic algorithm (GA) with binary chromosomes to search for optimal pump rates as well as optimal binary values of scheduling variables. Using the GA, the constraints are moved as the penalty terms to the objective function. Then, a multiobjective function is formulated:

where  $w_1$ ,  $w_2$ ,  $w_3$  are the weights to reflect the priorities, which in this study are in order of minimizing the concentration violation at Lula wells, minimizing the concentration violation in the remediation area, and minimizing the total amount of water injected and extracted.

# (2) Concentration Prediction using Bayesian Model Averaging under Uncertainty of Head Boundary Values and Variograms for Hydraulic Conductivity

The optimized joint operations are subject to the uncertainty of model structure that can cause large constraint violations. To assess the robustness of the optimized operations, this study introduces the Bayesian model averaging (BMA) (Hoeting et al. 1999) to obtain the predicted concentrations to evaluate the violations at Lula wells and in the remediation area.

Let  $\mathbf{M} = \{\mathbf{M}^{(p)}; p = 1, 2, ...\}$  be a set of saltwater intrusion simulation models based on different boundary values of ground water heads. Each simulation model may have different variogram models to estimate hydraulic conductivity, which is denoted as  $\mathbf{\theta} = \{\theta_q^{(p)}; q = 1, 2, ...\}$ . Given data  $\mathbf{D}$ , the expectation and covariance of chloride concentrations using multiple models can be obtained as follows:

$$E(\mathbf{C} \mid \mathbf{D}) = \sum_{n} \sum_{q} E(\mathbf{C} \mid \mathbf{M}^{(p)}, \boldsymbol{\theta}_{q}^{(p)}, \mathbf{D}) \Pr(\boldsymbol{\theta}_{q}^{(p)} \mid \mathbf{M}^{(p)}, \mathbf{D}) \Pr(\mathbf{M}^{(p)} \mid \mathbf{D}),$$
(6)

$$\operatorname{Cov}(\mathbf{C} \mid \mathbf{D}) = \operatorname{E}_{\mathbf{M}} \operatorname{E}_{\theta} \left[ \operatorname{Cov} \left[ \mathbf{C} \mid \mathbf{M}^{(p)}, \boldsymbol{\theta}_{q}^{(p)}, \mathbf{D} \right] \right] + \operatorname{E}_{\mathbf{M}} \operatorname{Cov}_{\theta} \left[ \operatorname{E} \left[ \mathbf{C} \mid \mathbf{M}^{(p)}, \boldsymbol{\theta}_{q}^{(p)}, \mathbf{D} \right] \right] + \operatorname{Cov}_{\mathbf{M}} \operatorname{E}_{\theta} \left[ \operatorname{E} \left[ \mathbf{C} \mid \mathbf{M}^{(p)}, \boldsymbol{\theta}_{q}^{(p)}, \mathbf{D} \right] \right]$$
(7)

where  $E_M E_\theta \Big[ Cov \Big[ \mathbf{C} \big| \mathbf{M}^{(p)}, \theta_q^{(p)}, \mathbf{D} \Big] \Big]$  is the within-covariance of concentration,  $E_M Cov_\theta \Big[ E \Big[ \mathbf{C} \big| \mathbf{M}^{(p)}, \theta_q^{(p)}, \mathbf{D} \Big] \Big]$  is the covariance of concentration due to different variogram models in simulation models, and  $Cov_M E_\theta \Big[ E \Big[ \mathbf{C} \big| \mathbf{M}^{(p)}, \theta_q^{(p)}, \mathbf{D} \Big] \Big]$  is the covariance of concentration due to different simulation models.  $Pr \Big( \mathbf{M}^{(p)} \big| \mathbf{D} \Big)$  is the posterior probability of simulation model p and  $Pr \Big( \theta_q^{(p)} \big| \mathbf{M}^{(p)}, \mathbf{D} \Big)$  is the posterior probability of variogram model p used in simulation model p.

The likelihood value,  $\Pr(\mathbf{D} | \mathbf{M}^{(p)}, \theta_q^{(p)})$ , is needed in order to calculate the posterior model probabilities and is approximated using the Bayesian information criterion (BIC) (Raftery 1995; Madigan et al. 1996):  $\Pr(\mathbf{D} | \mathbf{M}^{(p)}, \theta_q^{(p)}) \approx \exp(-\frac{1}{2} \operatorname{BIC}_q^{(p)})$ , where the BIC is

$$BIC_{q}^{(p)} = -2 \ln \Pr \left( \mathbf{D} \mid \mathbf{M}^{(p)}, \theta_{q}^{(p)}, \hat{\boldsymbol{\beta}}_{q}^{(p)} \right) + m_{q}^{(p)} \ln L.$$
 (8)

where  $\hat{\beta}_q^{(p)}$  is the maximum-likelihood estimated unknown parameters,  $m_q^{(p)}$  is the dimension of  $\hat{\beta}_q^{(p)}$ , and L is the size of the data  $\mathbf{D}$ . In this study,  $\beta_q^{(p)}$  refers to the data weighting coefficients in the GP methods used to estimate hydraulic conductivity (Tsai 2006).

Therefore, one can assess the constraint violations by using the BMA expectation for concentration prediction as follows

where  $C_{BMA}$  is obtained by Eq. (6) using the variance window (Tsai and Li 2008a,b), which is

$$C_{BMA} = \frac{\sum_{p} \sum_{q} E\left(C \mid \mathbf{M}^{(p)}, \boldsymbol{\theta}_{q}^{(p)}, \mathbf{D}\right) \exp\left(-\frac{1}{2}\alpha\Delta \operatorname{BIC}_{q}^{(p)}\right)}{\sum_{p} \sum_{q} \exp\left(-\frac{1}{2}\alpha\Delta \operatorname{BIC}_{q}^{(p)}\right)}.$$
(10)

This optimization problem is very time-consuming because it involves many simulation models and variogram models in the management model.

# **Principal Findings and Significance**

#### (1) Model Uncertainty

To assess the robustness of the optimized joint operations under this uncertainty, five groundwater flow models are created, which have 0%,  $\pm 10\%$ , and  $\pm 20\%$  changes of the predetermined head boundary values over the entire boundary. Moreover, the uncertainty in experimental variograms for hydraulic conductivity is also considered. Three variogram models (exponential (EXP), spherical (SPH) and Gaussian (Gau) models) are used. A total of 15 simulation models are developed. Detail information can be found in Tsai (2010).

#### (2) No-Action Scenario

Without the hydraulic barrier and extraction systems (no-action scenario), the chloride concentration is slowly moving northward toward the Lula wells. The concentration distributions predicted by the best model (Gau+0%) and the BMA are similar. Both confirm that the 2.5% isochlor does not reach the Lula wells within the management period. The variances of the predicted chloride concentrations due to different variogram models in individual simulation models are much smaller than the variances due to different simulation models. Given the similar weights of the variogram models in the best and second best simulation models, this indicates similar concentration predictions made by different variogram models within a simulation model. However, different simulation models due to head boundary uncertainty exhibit relatively large differences in concentration predictions.

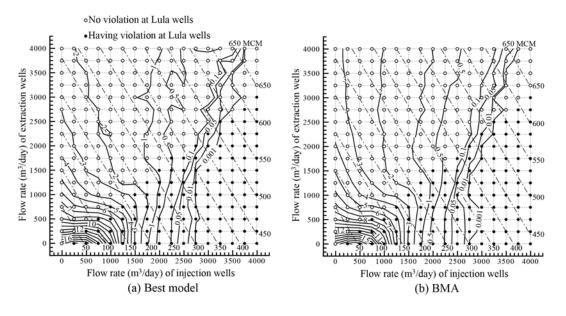
#### (3) Joint Operations with Well Pumps Active All Time

By considering the well pumps of the hydraulic barrier and extraction systems active all the time, the injection rates and extraction rates are increased systematically from the no-action scenario to illustrate the impact of the systems on the saltwater intrusion. A viable remedial action is defined for the case where the sum of violations (the second term in Eq. (5)) at the Lula wells is zero during the management period. Otherwise, the remedial action is not acceptable. For example, the no-action scenario is a viable scenario. A viable remediation scenario

represents the minimum requirement for an operation action because any actions that cause the Lula wells to be contaminated are not acceptable. Moreover, optimized operations would become very expensive if one was restricted to zero violation in the remediation area at the end of the management period. This study relaxes this restriction for practical purposes and considers a remedial action acceptable with a violation of less than 0.001 for the third term in Eq. (5). The threshold for this small acceptable violation is subjective and depends on decision makers.

Figure 2 shows the matrix of scenarios without violation (open circles) and with violation (filled circles) at Lula wells, created by enumerating the combinations of different injection rates and extraction rates using the best model (Gau+0) and the BMA. The injection and extraction rates are operated full time for 15 years. The total amount of pumped and injected water in million cubic meters (MCM) is plotted in Figure 2, which is the potential (maximum) amount of water the systems need to deal with. For example, an injection rate of 3250 m<sup>3</sup>/day and extraction rate of 2750 m<sup>3</sup>/day operate a potential amount of water of 537 MCM. Figure 2 also shows the contour lines of the sum of violations in the remediation area at the end of the 15-year management period. Based on the information in Figure 2, one can draw the following observations: (1) Actions with low injection rates with low extraction rates are unacceptable because they cannot cleanup the remediation area even though they are viable actions to the Lula wells. (2) High injection rates with low extraction rates are unacceptable because the hydraulic barrier system pushes northward and end up brackish water in the Lula wells. This can result in zero violation in the remediation area at the end of the management period. (3) Low injection rates with high extraction rates are generally not acceptable remedial actions. While no violation occurs in the Lula wells, the extraction system enlarges and deepens the depression core, induces more saltwater intrusion northward, and causes high violations in the remediation area. (4) Using higher injection rates and higher extraction rates is likely to achieve the goal of cleaning the brackish water in the remediation area without jeopardizing the Lula wells.

Figure 2: Matrix of actions with different injection and extraction rates using (a) the best model and (b) the BMA with the variance window. Open circle represents a viable action. Filled circle represents an unacceptable action. Solid line represents the sum of violations in the remediation area. The dotted-dashed line represents the total amount water injected and pumped in million cubic meters (MCM).



#### (4) Joint Operation Optimization

To reduce the complexity of the management model and increase the efficiency of searching for the optimal operation, the operation considers all injection wells and all pumping wells are active or inactive on a monthly basis for 15 years. Therefore, there are 180 scheduling variables for the injection wells and 180 scheduling variables for the pumping wells. A micro-GA solver (Carroll 1996) is used to minimize the objective function. The population in the micro-GA is five, the uniform crossover probability is 0.5, and the mutation probability is 0.02. The tournament selection strategy is used. The maximum number of generations for each GA run is 200. These GA parameters are suggested in the solver (Carroll 1996). The maximum injection rate  $(q_{max}^R)$  and extraction rate  $(q_{max}^P)$  in the GA are set to 4,000  $m^3/day$ . The injection rate and extraction rate are given the same length of 12 bits in the binary chromosomes. To prioritize the multiple objections, it sets  $w_1 = 10^{-11}$ ,  $w_2 = 100$ ,  $w_3 = 1.0$  for the objective function. The length of a binary chromosome is 384 bits. To obtain the fitness of each chromosome (one possible operation solution), the 12 simulation models are executed together to calculate the BMA concentrations. The computation is extremely extensive.

Again, the author recognizes the possibility of considering individual operations of the well pumps on the monthly basis. This will reduce operation costs by increasing flexibility in well operations in the management model. However, this will result in 3,600 scheduling variables for the injection wells and 2,160 scheduling variables for the pumping wells. This complicated optimization problem is avoided in this study.

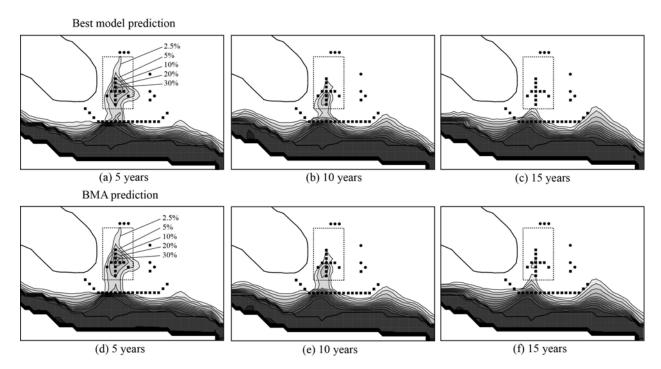
Two management models are compared to show the difference if model uncertainty is not considered. The first management model only considers the best model (Gau+0%). The second management model uses the BMA to predict concentration based on the 12 simulation models. Again, for the no-action scenario, two management models show no violation at Lula wells. However, the sum of violations in the remediation area is very high. If considering the best model only in the management model, the GA obtains the optimal injection rate to be 3,217  $m^3/day$  and the optimal extraction rate to be 2,448  $m^3/day$ . No violation occurs at the Lula wells and in the remediation area at the end of the management period. The total amount of water injected and pumped is 331 MCM. Comparing to the same injection and extraction rates in Figure 2, the management model significantly reduces concentration violations and the amount of water to deal with compared to pumping all wells all 15 years. Figure 3(a)-(c) shows the chloride concentration predictions at 5-years, 10-years, and 15-years. However, if model uncertainty is considered, one can test if the optimal operation from the best model is acceptable by re-evaluating the sum of violations using the BMA concentrations. This optimal solution produces noticeable violation in the remediation area at the end of the management period. The violation can be seen in Figure 3(f) at the end of the 15 years based on the BMA prediction. The violation is expected because the optimal operation from the best model neglects other good models and gives a biased solution.

Using the BMA to predict chloride concentration in the management model, the GA increases the optimal injection rate to  $3,729 \, m^3/day$  and increases the optimal extraction rate to  $3,012 \, m^3/day$  in order to reduce the violations from other models. The increased injection and extracting rates due to considering model uncertainty reflect the need of "overdesigning" the strategy to insure reliability (Wagner and Gorelick 1987). The optimal operation using the BMA presents an acceptable solution because no violation occurs at the Lula wells and the sum of violations in the remediation area is less than 0.001. The total amount of water injected and

pumped is 371 MCM. The optimal operation using BMA is also tested if it is an acceptable solution for the best model. After re-evaluating the sum of violations, the optimal operation using the BMA also works for the best model. The variances of chloride concentration at the end of 15 years due to different variogram models in individual simulation models are much smaller than the variances due to different simulation models.

Using the BMA prediction in the management model does not prevent other models from violation. An exhaustive management model can consider the constraints that include concentration predictions from individual models, but this would result in a very expensive management policy in terms of the total amount of injected and pumped water in order to satisfy all models. Moreover, this way would exaggerate the influence from insignificant models. With BMA, one can avoid this problem while considering the model uncertainty.

Figure 3: Isochlors predicted by the best model (Gau+0%) at (a) 5 years, (b) 10 years, and (c) 15 years, and by the BMA with the variance window at (d) 5 years, (e) 10 years, and (f) 15 years, given the optimal joint operation, injection rate =  $3.217 \text{ m}^3/\text{day}$  and extraction rate =  $2.448 \text{ m}^3/\text{day}$ , from the best model.



#### **Conclusions**

- [1] Groundwater management is far more difficult and complex because of model structure uncertainty. Uncertain model structure often results in multiple possible simulation models. Management plans under the consideration of a single simulation model tend to bias optimized operations. To alleviate the biasedness, a reliable groundwater management model should take into account the predictions from multiple simulations models.
- [2] Bayesian model averaging (BMA) has been shown to be capable of integrating multiple models for prediction in the management model. Optimized operations based on the BMA predictions show more reliable management outcomes than those from one simulation model. However, the optimized operation is more expensive in order to reduce constraint violations elevated by considering many models.

- [3] The study has demonstrated the importance of considering the model structure uncertainty in a real-world case study. Using the best model underestimates the optimized injection rate and extraction rate for the hydraulic barrier and extraction systems. Using the BMA prediction for chloride concentration, the optimized injection and extraction rates increase to reduce the concentration violation in the remediation area.
- [4] The study also demonstrates the importance of using the variance window for uncertainty analysis in the management model. Using Occam's window literally accepts only the best model and neglects model uncertainty. However, the incorporation of more simulation models in the management model, as suggested by the variance window, could result in more expensive operations in order to reduce additional constraint violations created by the additional simulation models. A further investigation should be conducted to understand the impact of the size of the variance window with respect to Occam's window.

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#### **Publications**

#### 1. Articles in Refereed Scientific Journals

- Tubbs, K. R., and F. T.-C. Tsai. (2010). GPU Accelerated Lattice Boltzmann Model for Shallow Water Flow and Mass Transport, International Journal for Numerical Methods in Engineering, DOI: 10.1002/nme.3066.
- Tsai, F. T.-C. and X. Li. (2010). Reply to Comment on "Inverse Groundwater Modeling for Hydraulic Conductivity Estimation Using Bayesian Model Averaging and Variance Window" by Ye, M., D. Liu, S.P. Neuman, and P.D. Meyer, Water Resources Research. doi:10.1029/2009WR008591.
- Servan-Camas, B., and F. T.-C. Tsai. (2010). Two-Relaxation-Time Lattice Boltzmann Method for Anisotropic Dispersive Henry Problem. Water Resources Research. doi:10.1029/2009WR007837.
- Tubbs, K. R., and F. T.-C. Tsai (2009). Multilayer Shallow Water Flow using Lattice Boltzmann Model with High Performance Computing. Advances in Water Resources, 32(11), 1767-1776.
- Li, X., and F. T.-C. Tsai.(2009). Bayesian Model Averaging for Groundwater Head Prediction and Uncertainty Analysis Using Multimodel and Multimethod. Water Resources Research, 45, W09403. doi:10.1029/2008WR007488.
- Tsai, F. T.-C., Bayesian Model Averaging Assessment on Groundwater Management under Model Structure Uncertainty, Stochastic Environmental Research and Risk Assessment, 24(6), 845-861, 2010.

#### 2. Book Chapter

• Tsai, F. T.-C. and W. W-G. Yeh. (2010). Chapter 7: Model Calibration and Parameter Structure Identification in Characterization of Ground Water Systems, in Ground Water Management Manual (M. Aral and S. Taylor ed.) American Society of Civil Engineers. Accepted.

#### 3. Dissertations

• Kevin R. Tubbs, 2010, Ph.D. Dissertation "Lattice Boltzmann Modeling for Shallow Water Equations Using High Performance Computing, Engineering Science Program, College of Engineering, Louisiana State University, Baton Rouge, Louisiana, 128 pages.

#### 4. Water Resources Research Institute Reports

• Frank Tsai, 2009, Saltwater Intrusion Management with Conjunctive Use of Surface Water and Ground Water, Louisiana Water Resources Research Institute, Louisiana State University, Baton Rouge, Louisiana, 10 pages. (USGS 104G)

• Frank Tsai, 2009, Electrical Resistivity Tomography (ERT) Laboratory Experiments on Saltwater Encroachment Tracking and Modeling in Saturated Heterogeneous Sediment, Louisiana Water Resources Research Institute, Louisiana State University, Baton Rouge, Louisiana, 10 pages. (USGS 104B)

# 5. Conference Proceedings

- Tsai, F. T.-C. (2010). A Co-Generalized Parameterization Method for Hydraulic Conductivity Estimation, World Water & Environmental Resources Congress, Providence, Rhode Island, May 16-20, 2010.
- Tsai, F. T.-C. (2010). Multimodel Approach for Groundwater Model Calibration, Prediction, and Application, World Water & Environmental Resources Congress, Providence, Rhode Island, May 16-20, 2010. (invited)

# 6. Other Publications (Abstract/Presentations)

- Tsai, F. T.-C. (2010). Data Fusion using Co-Generalized Parameterization: Hydraulic Conductivity Estimation, 2010 Western Pacific Geophysics Meeting, 22–25 June 2010, Taipei, Taiwan
- Tsai, F. T.-C. (2010). Hierarchical Bayesian Model Averaging for Groundwater Multimodel Prediction and Management under Uncertainty, 2010 Western Pacific Geophysics Meeting, 22–25 June 2010, Taipei, Taiwan
- Tsai, F. T.-C. (2010). A Co-Generalized Parameterization Method for Hydraulic Conductivity Estimation, World Water & Environmental Resources Congress, Providence, Rhode Island, May 16-20, 2010.
- Tsai, F. T.-C. (2010). Multimodel Approach for Groundwater Model Calibration, Prediction, and Application, World Water & Environmental Resources Congress, Providence, Rhode Island, May 16-20, 2010. (invited)

#### 7. Student Support

• Kevin R. Tubbs, PhD, Spring 2010 (Graduated)